PS Algorithms and Data Structures 2026

Task sheet 7

Task 19

Consider a hash table of size m using chaining for collision resolution. After inserting n elements, the expected length of each chain is $\frac{n}{m}$. This assumes simple uniform hashing.

Show that: For any constant $c > 0$, the probability that a chain contains at least $c \cdot \frac{n}{m}$ elements is at most $\frac{1}{c}$. In doing so, you apply Markov's inequality and prove it.

Task 20

Let G be a directed graph represented in (a) adjacency list format and (b) adjacency matrix format. For each of the following problems, determine the tight runtime complexity in Θ-notation based on the following parameters: (1) number of vertices n, (2) number of edges m, (3) out-degree of a vertex $\deg^+(v)$, (4) maximum out-degree Δ^+, (5) in-degree of a vertex $\deg^-(v)$, and (6) maximum in-degree Δ^-

Choose the corresponding complexity for both (a) and (b) and briefly justify your answer.

Computation of the out-degree of a vertex v	$\Theta(1)$	$\Theta(\deg^+(v))$	$\Theta(\deg^-(v))$	$\Theta(n)$	$\Theta(m)$
Computation of the in-degree of a vertex v	$\Theta(1)$	$\Theta(\deg^+(v))$	$\Theta(\deg^-(v))$	$\Theta(n)$	$\Theta(m)$
Check for existence of an edge between two given vertices u, v	$\Theta(1)$	$\Theta(\deg^+(v))$	$\Theta(\deg^-(v))$	$\Theta(n)$	$\Theta(m)$
Check for existence of a 2-cycle starting at vertex v	$\Theta(n)$	$\Theta(\deg^+(v)\Delta^+)$	$\Theta(\deg^-(v)n)$	$\Theta(n^2)$	$\Theta(nm)$

A 2-cycle is a cycle of length 2 containing two vertices u and v such that $(u, v) \in E$ and $(v, u) \in E$.

Task 21

Two nodes u and v in a graph are called *connected* if in the graph (1) there is a path from u to v and (2) a path from v to u. In undirected graphs, statements (1) and (2) are equivalent.

Develop an algorithm with runtime $O(|V| + |E|)$ that assigns an (integer) label $v.\ell$ to each vertex v of a given undirected graph $G = (V, E)$ such that $v.\ell = u.\ell$ for every pair of vertices u and v if and only if u and v are connected.