PS Algorithms and Data Structures 2024

Task sheet 9

Task 25

Why must the edge weights be nonnegative for Dijkstra's algorithm to give a correct result?

Give an example graph with negative edge weights, but without a negative cycle for which Dijkstra's algorithm does not work correctly.

Task 26

Let $G = (V, E)$ be a directed, weighted graph with edge weight $w(u, v) \in \mathbb{R}$ for each edge $(u, v) \in E$. Furthermore, let f be a function that assigns a number $f(v) \in \mathbb{R}$ to each node $v \in V$. In addition, let G' be the graph that contains the same nodes and edges as G and in which each edge $(u, v) \in E$ receives the weight $w'(u, v) = w(u, v) + f(u) - f(v)$. Show that G and G' contain the same shortest paths.

Attention: The distances may very well change.

Task 27

It is given a directed, weighted graph $G = (V, E)$ (with positive edge weights) and a subset of nodes $U \subseteq V$. Develop an algorithm with runtime $O((|E|+|V|) \log |V|)$ that calculates for each node $v \in V \setminus U$ the node $u^* \in U$ with the smallest distance to v, i.e., $u^* := \arg\min_{u \in U} \delta(u, v)$.